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Phase-lag and diffusion in porous thermoelastic micropolar media with initial pressure and rotational forces affected by modified Ohm's law and gravity

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ABSTRACT

This study explores the diffusion in micropolar thermoelastic materials with voids rotating at a uniform angular velocity. Researchers utilized the harmonic solution as an analytical method to examine the micropolar porous medium affected by gravity and rotation with thermal shock in descriptive equations with the initial mechanical pressure. Numerical calculations were conducted, and results were graphically displayed for displacement, stresses, temperature, change in volume fraction field, microrotation vector, chemical concentration, and chemical potential for two dual-phase-lag parameter values. The study's findings have implications in seismic engineering and manufacturing.

KEYWORDS: Diffusion, Gravity, Modified Ohm's law, Micropolar and Porous thermoelastic, Phase-lag, Rotation, Temperature gradient.

1. INTRODUCTION

The theory of elastic materials with voids in thermoelastic materials is an intriguing extension of classical elasticity theory. This concept was introduced and developed as [1-3]. Othman and Hilal [4] studied the impact of gravity and rotation in thermoelastic materials with voids. Various authors [5-12] have discussed many topics concerning heat and mass transfer. The micropolar theory assumes that a material's reaction is affected by the motion of its internal structure considering the translational degrees of freedom of material points as Eringen [13]. Thermodiffusion in elastic solids is the random movement of particles from areas of high concentration as in Nowacki [14]. The heat conduction equation is based on the Fourier law of heat equation. This equation was justified in more form to describe the wave propagation of the heat wave as references [15-17]. Recently, Tzou [18-20] introduced the Dual-Phase-Lag model, which explains the interactions between phonons and electrons at the microscopic level of the medium by the heat flux τ_q phase lag and the temperature gradient τ_θ phase lag. In this theory, the heat equation is the hyperbolic type and predicts the finite speed of thermal wave propagation.

Rotation of an elastic medium taking Coriolis and centrifugal accelerations introduced by Schoenberg and Censor [21]. The initial stresses and the gravity in solids have significant effects on their response to applied loads. Ames and Straughan [22] presented the results for the initial stress on thermoelastic bodies. Bromwich [23] established the influence of gravity on elastic waves. Montanaro [24] studied the initial pressure behavior in solids. The magnetic field effects in thermoelastic solids are useful due to their applications in plasma physics and other topics. More motivating thermoelastic problems were investigated in [25-52].

The present paper investigates the diffusion and modified Ohm's law in a micropolar thermoelastic medium with voids and rotation. The harmonic solution is an analytical method used to get solutions of the descriptive functions in the case of the two values of the heat flux and the temperature gradient while the medium is affected by gravity and rotation with thermal shock. The material underwent initial mechanical pressure. Numerical calculations were conducted, and results were graphically displayed for the functions for two dual-phase-lag parameter values. The study's findings have implications in seismic engineering and manufacturing.

2. BASIC EQUATIONS AND SOLUTION WITH BOUNDARY CONDITIONS

Due to Ieşan [4], Eringen [15], Schoenberg and Censor [22], and Montanaro [24]. The field equations for isotropic, linear, homogenous, rotator micropolar thermoelastic material with voids, and pre-stressed in a magnetic field and the gravity are

$$\sigma_{ij,i} + F_i = \rho[u_{i,tt} + (\Omega_i \times (\Omega_i \times u_i)) + (2\Omega_i \times u_{i,t})], \quad (1)$$

$$\omega_0 \psi_{,ii} - \xi_1 \psi - b\,\psi_{i,i} + r\,T = \chi\,\rho\psi_{i,tt}, \quad (2)$$

$$m_{ij,i} + \varepsilon_{ijr}\,\sigma_{ir} = J_0\,\rho[\phi_{i,tt} + (\Omega \times \phi_{i,t})], \quad (3)$$

$$d\,\beta_2\,e_{,ii} + d\,a^* T_{,ii} + d\,b_1 C_{,ii} + C_{,t} = 0, \quad (4)$$

The heat equation as Tzou [18] and the Lorentz's force equations were taken as Lorentz [53]

$$k\,(1+\tau_\theta \frac{\partial}{\partial t})T_{,ii} - r\,T_0(1+\tau_q \frac{\partial}{\partial t})\psi = (1+\tau_q \frac{\partial}{\partial t})(\rho C_E\,T_{,t} + \beta_1 T_0\,e_{,t}), \quad (5)$$

$$\sigma_{ij} = \lambda u_{r,r}\,\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \bar{k}\,(u_{j,i} - \varepsilon_{ijr}\,\phi_r) + b\,\psi\,\delta_{ij} - \beta_1 T\,\delta_{ij} - p^*(\omega_{ij} + \delta_{ij}), \quad (6)$$

$$m_{ij} = \alpha\,\phi_{r,r}\,\delta_{ij} + \beta\,\phi_{i,j} + \gamma\,\phi_{j,i}, \quad (7)$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),\ \omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}),\ \beta_1 = (3\lambda + 2\mu + \bar{k})\alpha_t,\ \beta_2 = (3\lambda + 2\mu + \bar{k})\alpha_c, \quad (8)$$

$$\rho\,T_0\,S = \rho\,C_E\,T + \beta_1\,T_0\,e_{kk} + a^*\,T_0\,C, \quad (9)$$

$$P = \beta_2\,e_{kk} + b_1 C - a^* T, \quad (10)$$

$$T = \theta - T_0,\quad \left|\frac{T - T_0}{T_0}\right| << 1, \quad (11)$$

$$\nabla \times h_i = J_i + \frac{\partial D_i}{\partial t},\qquad \nabla \times E_i = -\frac{\partial B_i}{\partial t}, \quad (12)$$

$$\nabla .\ B_i = 0,\qquad \nabla .\ D_i = \rho_e,\qquad E_i = -\mu_0\,(\frac{\partial u_i}{\partial t} \times H_i), \quad (13)$$

$$D_i = \varepsilon_0\,E_i,\qquad B_i = \mu_0\,(\mathrm{H}_0 + h_i),\qquad F_i = \mu_0\,(J_i \times H_i), \quad (14)$$

$$J_i = \sigma_0[E_i + \mu_0 \frac{\partial u_i}{\partial t} \times H_i] - k_0\,\nabla T, \quad (15)$$

With the abbreviations

Symbol	Name	Symbol	Name
λ, μ	Lamé constants	ρ	the density

α, β, γ and $\bar{k}$	the micropolar constants	α_c	diffusion expansion coefficient
ω_0, ξ_1, b, r, and χ	the voids constants	C_E	the specific heat
ψ	change in the volume fraction field	k	the thermal conductivity
α_t	the thermal expansion coefficient	T_0	the reference temperature
ϕ_i	the microrotation vector	σ_{ij}	the stress components
e_{ij}	the cubic strain (dilation)	Ω_i	the angular velocity
m_{ij}	the couple stresses	J_0	the microinertia
F_i	the Lorentz force	E_i	the induced electric field
h_i	the induced magnetic field	D_i	the electric displacement vector
J_i	the current density vector	H_0	the uniform magnetic field
ε_0	the electric permittivity	μ_0	the magnetic permeability
σ_0	the electric conductivity	ρ_e	the charge density
k_0	the connection of the temperature gradient and the electric current density		

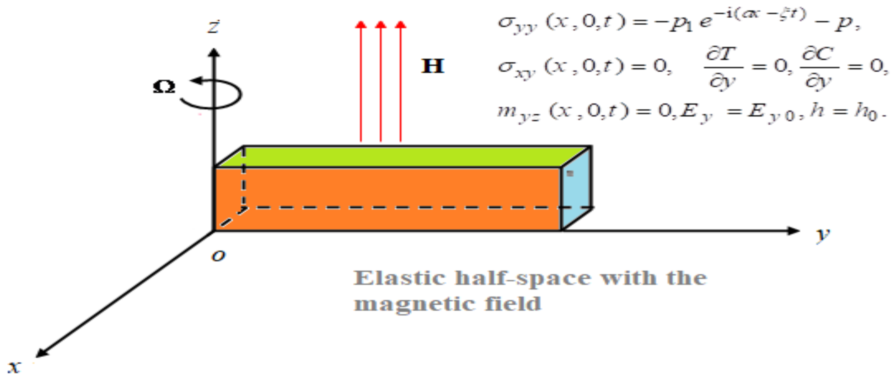

Fig. 1: Geometry of the problem with boundary conditions

To express the 2-D problem formulation with the Bromwich equations as [23] and the dimensionless variables in the following form

$$x_i' = \frac{\omega_1^*}{c_0} x_i, \quad u_i' = \frac{\rho c_0 \omega_1^*}{\beta_1 T_0} u_i, \quad \phi_z' = \frac{\rho c_0^2}{\beta_1 T_0} \phi_z, \quad \psi' = \frac{\rho \chi \omega_1^{*2}}{\beta_1 T_0} \psi, \quad \Omega' = \frac{\Omega}{\omega_1^*}, \quad T' = \frac{T}{T_0}, \quad g' = \frac{g}{\omega_1^* c_0}, \quad m_{ij}' = \frac{\omega_1^*}{\beta_1 c_0 T_0} m_{ij},$$

$$(t', \tau_\theta', \tau_q') = \omega_1^* (t, \tau_\theta, \tau_q), \qquad (\sigma_{ij}', p_1') = \frac{1}{\beta_1 T_0} (\sigma_{ij}, p_1), \qquad k_0' = \frac{\mu_0 H_0}{\beta_1} k_0, \qquad h' = \frac{\omega_1^*}{\mu_0 \sigma_0 H_0} h,$$

$$E_i' = \frac{\omega_1^* c_0}{\mu_0^2 \sigma_0 H_0} E_i, \quad C' = \frac{\beta_2}{\beta_1 T_0} C, \quad P' = \frac{P}{\beta_2}, \quad C^2 = \frac{1}{\mu_0 \varepsilon_0}, \quad \omega_1^* = \frac{\rho C_E c_0^2}{k}, \quad c_0^2 = (\frac{\lambda + 2\mu + \bar{k}}{\rho}). \qquad (16)$$

Then the field equations (neglecting the prime) will be

$$\nabla^2 u + a_1 \frac{\partial e}{\partial x} + a_2 \frac{\partial \phi_z}{\partial y} + a_3 \frac{\partial \psi}{\partial x} - a_4 \frac{\partial}{\partial x}(T - C) + a_5 E_y - a_6 \frac{\partial u}{\partial t}$$
$$+ a_4 \frac{\partial v}{\partial x} - a_7 \frac{\partial T}{\partial y} = a_4 [\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega \frac{\partial v}{\partial t}], \quad (17)$$

$$\nabla^2 v + a_1 \frac{\partial e}{\partial y} - a_2 \frac{\partial \phi_z}{\partial x} + a_3 \frac{\partial \psi}{\partial y} - a_4 \frac{\partial}{\partial y}(T - C) - a_5 E_x - a_6 \frac{\partial v}{\partial t}$$
$$- a_4 \frac{\partial u}{\partial x} + a_7 \frac{\partial T}{\partial x} = a_4 [\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega \frac{\partial u}{\partial t}], \quad (18)$$

$$\nabla^2 \phi_z - a_8 \phi_z + a_9 (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) - a_{10} \frac{\partial^2 \phi_z}{\partial t^2} = 0, \quad (19)$$

$$\nabla^2 \psi - a_{11} \psi - a_{12} (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + a_{13} T - a_{14} \frac{\partial^2 \psi}{\partial t^2} = 0, \quad (20)$$

$$\nabla^2 C + a_{15} \nabla^2 (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + a_{16} \nabla^2 T + a_{17} \frac{\partial C}{\partial t} = 0, \quad (21)$$

$$(1 + \tau_\theta \frac{\partial}{\partial t}) \nabla^2 T - a_{18} (1 + \tau_q \frac{\partial}{\partial t}) \psi - a_{19} (1 + \tau_q \frac{\partial}{\partial t}) \frac{\partial T}{\partial t} - a_{20} \frac{\partial}{\partial t} (1 + \tau_q \frac{\partial}{\partial t}) (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0, \quad (22)$$

Using the Helmholtz theorem for the potential functions $\Pi(x, y, t)$ and $\Theta_i (x, y, t)$ as $u_i = \nabla \Pi + \nabla \times \Theta_i$, then

$$u = \frac{\partial \Pi}{\partial x} + \frac{\partial \Theta}{\partial y}, \quad \text{and} \quad v = \frac{\partial \Pi}{\partial y} - \frac{\partial \Theta}{\partial x}, \quad (23)$$

The governing equations will be

$$\left[(a_1 + 1) \nabla^2 - (\frac{a_5 \, a33}{a_{35} (a_{32} + \varepsilon_1 \frac{\partial}{\partial t})} - a_6) \frac{\partial}{\partial t} + a_4 (\Omega^2 - \frac{\partial^2}{\partial t^2}) \right] \Pi - \left[2 a_4 \Omega \frac{\partial}{\partial t} + a_4 \frac{\partial}{\partial x} \right] \Theta + \left[\frac{a_5}{a_{35} (a_{32} + \varepsilon_1 \frac{\partial}{\partial t})} \right] h + [a_3] \psi - [a_4] C - [a_4] T = 0, \quad (24)$$

$$\left[2\Omega a_4 \frac{\partial}{\partial t} + a_4 \frac{\partial}{\partial x} \right] \Pi + \left[\nabla^2 + (\frac{a_5 \, a_{33}}{a_{35} (a_{32} + \varepsilon_1 \frac{\partial}{\partial t})} - a_6) \frac{\partial}{\partial t} - a_4 (\Omega^2 + \frac{\partial^2}{\partial t^2}) \right] \Theta + [a_2] \phi_z + \left[(\frac{a_5 \, a_{33}}{a_{35} (a_{32} + \varepsilon_1 \frac{\partial}{\partial t})}) - a_7 \right] T = 0, \quad (25)$$

$$\left[\frac{-a_{33}}{a_{35}} \frac{\partial}{\partial t} \nabla^2 \right] \Pi + \left[\nabla^2 - \varepsilon_1 \frac{\partial^2}{\partial t^2} - \frac{a_{32}}{a_{35}} \frac{\partial}{\partial t} \right] h = 0, \quad (26)$$

$$\left[-a_9 \nabla^2 \right] \Theta + \left[\nabla^2 - a_{10} \frac{\partial^2}{\partial t^2} - a_8 \right] \phi_z = 0, \quad (27)$$

$$\left[-a_{12} \nabla^2 \right] \Pi + \left[\nabla^2 - a_{14} \frac{\partial^2}{\partial t^2} - a_{11} \right] \psi + [a_{13}] T = 0, \quad (28)$$

$$\left[a_{15} \nabla^4 \right] \Pi + \left[\nabla^2 + a_{17} \frac{\partial}{\partial t} - a_{11} \right] C + \left[a_{16} \nabla^2 \right] T = 0, \quad (29)$$

$$\left[-a_{20} (1 + \tau_q \frac{\partial}{\partial t}) \frac{\partial}{\partial t} \nabla^2 \right] \Pi - \left[a_{18} (1 + \tau_q \frac{\partial}{\partial t}) \right] \psi + \left[(1 + \tau_\theta \frac{\partial}{\partial t}) \nabla^2 - a_{19} (1 + \tau_q \frac{\partial}{\partial t}) \frac{\partial}{\partial t} \right] T = 0, \quad (30)$$

where $a_1 = \dfrac{(\lambda+\mu+P^*/2)}{(\mu+\bar{k}-P^*/2)}$, $a_2 = \dfrac{\bar{k}}{(\mu+\bar{k}-P^*/2)}$, $a_3 = \dfrac{b\,c_0^2}{\chi\,\omega_1^{*2}\,(\mu+\bar{k}-P^*/2)}$, $a_4 = \dfrac{\rho c_0^2}{(\mu+\bar{k}-P^*/2)}$,
$a_5 = \dfrac{\rho\, c_0^2 \sigma_0^2\, \mu_0^2\, H_0^3}{\beta_1 T_0\, \omega_1^{*3}(\mu+\bar{k}-P^*/2)}$, $a_6 = \dfrac{\sigma_0\, c_0^2\, \mu_0^2\, H_0^2}{\omega_1^*(\mu+\bar{k}-P^*/2)}$, $a_7 = \dfrac{\rho c_0^2\, k_0}{\mu_0\, H_0(\mu+\bar{k}-P^*/2)}$, $a_8 = \dfrac{2\bar{k}\, c_0^2}{\gamma\,\omega_1^{*2}}$,
$a_9 = \dfrac{\rho c_0^2(\bar{k}-p^*)}{\gamma\,\beta_1 T_0}$, $a_{10} = \dfrac{\rho J_0 c_0^2}{\gamma}$, $a_{11} = \dfrac{\xi_1 c_0^2}{\omega_0}$, $a_{12} = \dfrac{b\,\chi}{\omega_0}$, $a_{13} = \dfrac{\rho\, c_0^2\, r\,\chi}{\beta_1\,\omega_0}$, $a_{14} = \dfrac{\rho\,\chi\, c_0^2}{\omega_0}$, $a_{15} = \dfrac{\beta_2^2}{\rho\, b_1\, c_0^2}$,
$a_{16} = \dfrac{a^*\beta_2}{\beta_1\, b_1}$, $a_{17} = \dfrac{1}{d\, b_1\,\omega_1^*\, c_0^2}$, $a_{18} = \dfrac{r\,\beta_1 T_0\, c_0^2}{\rho\,\chi\, k\,\omega_1^{*4}}$, $a_{19} = \dfrac{\rho\, C_E\, c_0^2}{k\,\omega_1^*}$, $a_{20} = \dfrac{\beta_1^2}{\rho\, k\,\omega_1^*}$, $a_{21} = \dfrac{\lambda}{\rho\, c_0^2}$, $a_{22} = \dfrac{1}{\rho\, c_0^2}$,
$a_{23} = \dfrac{2\mu+\bar{k}}{\rho\, c_0^2}$, $a_{24} = \dfrac{b}{\rho\,\chi\,\omega_1^{*2}}$, $a_{25} = \dfrac{\mu+(p^*/2)}{\rho\, c_0^2}$, $a_{26} = \dfrac{\mu+\bar{k}-(p^*/2)}{\rho\, c_0^2}$, $a_{27} = \dfrac{\bar{k}}{\rho\, c_0^2}$, $a_{28} = \dfrac{\gamma\,\omega_1^{*2}}{\rho\, c_0^4}$,
$a_{29} = \dfrac{-\beta_1 T_0}{\rho\, c_0^2}$, $a_{30} = \dfrac{-\beta_1 T_0}{\beta_2^2}$, $a_{31} = \dfrac{-a^* T_0}{\beta^2}$.

$$\frac{\partial h}{\partial y} = \sigma_0\,(E_x + \mu_0 H_0 \frac{\partial v}{\partial t}) + \varepsilon_0 \frac{\partial E_x}{\partial t} - k_0 \frac{\partial T}{\partial x}, \quad \text{and} \quad \frac{\partial h}{\partial x} = -\sigma_0\,(E_y - \mu_0 H_0 \frac{\partial u}{\partial t}) - \varepsilon_0 \frac{\partial E_y}{\partial t} + k_0 \frac{\partial T}{\partial y}, \tag{31}$$

$$\mu_0 \frac{\partial h}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}. \tag{32}$$

Which can be simplified as

$$-(a_{32} + \varepsilon_1 \frac{\partial}{\partial t})(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}) = \nabla^2 h - a_{33} \frac{\partial}{\partial t}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}), \tag{33-a}$$

$$(a_{32} + \varepsilon_1 \frac{\partial}{\partial t})(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}) = a_{34}\nabla^2 T - a_{33} \frac{\partial}{\partial t}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}), \tag{33-b}$$

$$\nabla^2 h - \frac{a_{32}}{a_{35}}\frac{\partial h}{\partial t} - \varepsilon_1 \frac{\partial^2 h}{\partial t^2} - \frac{a_{33}}{a_{35}}\frac{\partial}{\partial t}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0, \tag{33-c}$$

where $a_{32} = \dfrac{\sigma_0\,\mu_0}{\omega_1^*}$, $a_{33} = \dfrac{\beta_1 T_0}{\rho}$, $a_{34} = \dfrac{\beta_1 T_0\, k_0\,\omega_1^*}{\sigma_0\,\mu_0^2 H_0^2}$, $a_{35} = \dfrac{1}{c_0^2}$, $\varepsilon_1 = \dfrac{c_0^2}{c^2}$, $c^2 = \dfrac{1}{\mu_0\,\varepsilon_0}$, and c is the light's speed.

The harmonic wave solution of the obtained system can be supposed as

$$\{\Pi,\Theta,h,\phi_z,\psi,C,T\}(x,y,t) = \{\bar{\Pi},\bar{\Theta},\bar{h},\bar{\phi}_z,\bar{\psi},\bar{C},\bar{T}\}(y)\,\exp[\mathrm{i}(a x - \xi t)], \tag{34}$$

where $\{\bar{\Pi},\bar{\Theta},\bar{h},\bar{\phi}_z,\bar{\psi},\bar{C},\bar{T}\}(y)$ are the amplitudes of functions, a is the wave number, and ξ is the angular frequency.

The descriptive equations will be

$$\left[\mathrm{D}^2 - m_2\right]\bar{\Pi} + m_3\bar{\Theta} + m_4\bar{h} + m_5\,\bar{\psi} - m_6(\bar{C} - \bar{T}) = 0, \tag{35}$$

$$[m_7]\bar{\Pi} + \left[\mathrm{D}^2 - m_8\right]\bar{\Theta} + a_2\,\bar{\phi}_z + m_9\,\bar{T} = 0, \tag{36}$$

$$m_{10}\left[\mathrm{D}^2 - a^2\right]\bar{\Pi} + \left[\mathrm{D}^2 - m_{11}\right]\bar{h} = 0, \tag{37}$$

$$-a_9\left[\mathrm{D}^2 - a^2\right]\bar{\Theta} + \left[\mathrm{D}^2 - m_{12}\right]\bar{\phi}_z = 0, \tag{38}$$

$$-a_{12}\left[\mathrm{D}^2 - a^2\right]\bar{\Pi} + \left[\mathrm{D}^2 - m_{13}\right]\bar{\psi} + [a_{13}]\bar{T} = 0, \tag{39}$$

$$a_{15}\left[[D^2-a^2][D^2-a^2]\right]\bar{\Pi}+\left[D^2-m_{14}\right]\bar{C}+a_{16}\left[D^2-a^2\right]\bar{T}=0, \tag{40}$$

$$m_{15}\left[D^2-a^2\right]\bar{\Pi}-\left[m_{16}\right]\bar{\psi}+\left[D^2-m_{17}\right]\bar{T}=0, \tag{41}$$

where $m_1=a_1+1,\quad m_2=a^2-\frac{i\xi}{m_1}\left(\frac{a_5 a_{33}}{a_{35}(a_{32}-i\xi\varepsilon_1)}-a_6\right)-\frac{a_4}{m_1}(\Omega^2-\xi^2),\quad m_3=\frac{2i\xi\Omega a_4+ia\,a_4}{m_1},$

$m_4=\frac{a_5}{m_1 a_{35}(a_{32}-i\xi\varepsilon_1)},\quad m_5=\frac{a_3}{m_1},\quad m_6=\frac{a_4}{m_1},\quad m_7=-2i\xi\Omega a_4++ia\,a_4,$

$m_8=a^2+i\xi\left(\frac{a_5 a_{33}}{(a_{32}-i\xi\varepsilon_1)}-a_6\right)+a_4(\xi^2-\Omega^2),\quad m_9=\frac{a_5 a_{34}}{(a_{32}-i\xi\varepsilon_1)}-a_7,\quad m_{10}=\frac{i\xi a_{33}}{a_{35}},$

$m_{11}=a^2-\varepsilon_1\xi^2-\frac{i\xi a_{32}}{a_{35}},\quad m_{12}=a^2-a_{10}\xi^2+a_8,\quad m_{13}=a^2-a_{14}\xi^2+a_{11},\quad m_{14}=a^2+i\xi a_{17},$

$m_{15}=\frac{i\xi a_{20}(1-i\xi\tau_q)}{(1-i\xi\tau_\theta)},\quad m_{16}=\frac{a_{18}(1-i\xi\tau_q)}{(1-i\xi\tau_\theta)},\quad m_{17}=a^2-\frac{i\xi a_{19}(1-i\xi\tau_q)}{(1-i\xi\tau_\theta)}.$

Solving the system of the seven equations; (35)-(41) simultaneously yields:

$$\left[\left(\frac{d^2}{dy^2}\right)^7-\alpha_1\left(\frac{d^2}{dy^2}\right)^6+\alpha_2\left(\frac{d^2}{dy^2}\right)^5-\alpha_3\left(\frac{d^2}{dy^2}\right)^4+\alpha_4\left(\frac{d^2}{dy^2}\right)^3-\alpha_5\left(\frac{d^2}{dy^2}\right)^2+\alpha_6\left(\frac{d^2}{dy^2}\right)-\alpha_7\right]\left\{\bar{\Pi},\bar{\Theta},\bar{h},\bar{\phi}_z,\bar{\psi},\bar{C},\bar{T}\right\}(y)=0, \tag{42}$$

Since $\alpha_1,\ldots,\alpha_7$ are the material constants. The ordinary differential equation (42) gives 14 roots. To avoid the unbounded solutions, ignoring the positive roots, considering the negative roots only $(k_n, n=1,\ldots,7)$.

Solutions of the considered functions will be

$$\Pi(x,y,t)=\sum_{n=1}^{7}H_n\exp\{(-k_n y)+i(ax-\xi t)\},\qquad \Theta(x,y,t)=\sum_{n=1}^{7}H_n A_{1n}\exp\{(-k_n y)+i(ax-\xi t)\}, \tag{43}$$

$$h(x,y,t)=\sum_{n=1}^{7}H_n A_{2n}\exp\{(-k_n y)+i(ax-\xi t)\},\qquad \phi_z(x,y,t)=\sum_{n=1}^{7}H_n A_{3n}\exp\{(-k_n y)+i(ax-\xi t)\}, \tag{44}$$

$$\psi(x,y,t)=\sum_{n=1}^{7}H_n A_{4n}\exp\{(-k_n y)+i(ax-\xi t)\},\qquad C(x,y,t)=\sum_{n=1}^{7}H_n A_{5n}\exp\{(-k_n y)+i(ax-\xi t)\}, \tag{45}$$

$$T(x,y,t)=\sum_{n=1}^{7}H_n A_{6n}\exp\{(-k_n y)+i(ax-\xi t)\},\qquad u(x,y,t)=\sum_{n=1}^{7}H_n A_{7n}\exp\{(-k_n y)+i(ax-\xi t)\}, \tag{46}$$

$$v(x,y,t)=\sum_{n=1}^{7}H_n A_{8n}\exp\{(-k_n y)+i(ax-\xi t)\},\qquad \sigma_{xx}(x,y,t)=\sum_{n=1}^{7}H_n A_{9n}\exp\{(-k_n y)+i(ax-\xi t)\}, \tag{47}$$

$$\sigma_{yy}(x,y,t)=\sum_{n=1}^{7}H_n A_{10n}\exp\{(-k_n y)+i(ax-\xi t)\},\qquad \sigma_{zz}(x,y,t)=\sum_{n=1}^{7}H_n A_{11n}\exp\{(-k_n y)+i(ax-\xi t)\}, \tag{48}$$

$$\sigma_{xy}(x,y,t)=\sum_{n=1}^{7}H_n A_{12n}\exp\{(-k_n y)+\mathrm{i}(ax-\xi t)\},\qquad m_{yz}(x,y,t)=\sum_{n=1}^{7}H_n A_{14n}\exp\{(-k_n y)+\mathrm{i}(ax-\xi t)\},\tag{49}$$

$$m_{xz}(x,y,t)=\sum_{n=1}^{7}H_n A_{15n}\exp\{(-k_n y)+\mathrm{i}(ax-\xi t)\},\qquad P(x,y,t)=\sum_{n=1}^{7}H_n A_{16n}\exp\{(-k_n y)+\mathrm{i}(ax-\xi t)\},\tag{50}$$

$$E_x(x,y,t)=\sum_{n=1}^{7}H_n B_{17n}\exp\{(-k_n y)+\mathrm{i}(ax-\xi t)\},\qquad E_y(x,y,t)=\sum_{n=1}^{7}H_n B_{18n}\exp\{(-k_n y)+\mathrm{i}(ax-\xi t)\},\tag{51}$$

where

$$A_{1n}=\frac{-k_n^2+m_2-m_4A_{2n}-m_5A_{4n}+m_6A_{5n}\,m_6A_{6n}}{m_3},\quad A_{2n}=\frac{-m_{10}(k_n^2-a^2)}{(k_n^2-m_{11})},\quad A_{3n}=\frac{a_9A_{1n}(k_n^2-a^2)}{(k_n^2-m_{12})},$$

$$A_{4n}=\frac{a_{12}k_n^4-(a_{12}m_{17}+a_{12}a^2-a_{12}m_{15})k_n^2+(m_{17}a_{12}-a_{13}m_{15})a^2}{k_n^4-(m_{13}+m_{17})k_n^2-(m_{13}m_{17}+a_{13}m_{16})},$$

$$A_{5n}=\frac{a_{15}k_n^4-(2a_{15}a^2-a_{16}A_{6n})k_n^2+(a_{15}a^4-a_{16}A_{6n}a^2)}{(k_n^2-m_{14})},\qquad A_{6n}=\frac{m_{16}A_{4n}-m_{15}(k_n^2-a^2)k_n^2}{(k_n^2-m_{17})},$$

$$A_{7n}=\mathrm{i}a-k_nA_{1n},\qquad A_{8n}=-k_n-\mathrm{i}aA_{1n},$$

$$A_{9n}=a_{21}(\mathrm{i}aA_{7n}-k_nA_{8n})+\mathrm{i}aa_{22}A_{7n}+a_{23}A_{4n}-A_{5n}-A_{6n},$$

$$A_{10n}=a_{21}(\mathrm{i}aA_{7n}-k_nA_{8n})-a_{22}k_nA_{8n}+a_{23}A_{4n}-A_{5n}-A_{6n},$$

$$A_{11n}=a_{21}(\mathrm{i}aA_{7n}-k_nA_{8n})+a_{23}A_{4n}-A_{5n}-A_{6n},$$

$$A_{12n}=-a_{25}k_nA_{7n}+\mathrm{i}aa_{26}A_{8n}-a_{27}A_{3n},$$

$$A_{14n}=-a_{28}k_nA_{3n},\quad A_{15n}=\mathrm{i}aa_{28}A_{3n},\ A_{16n}=a_{29}(\mathrm{i}aA_{7n}-k_nA_{8n})+a_{30}A_{5n}+a_{31}A_{6n},\qquad \xi=b_1+\mathrm{i}b_2.$$

$$A_{17n}=\frac{1}{(a_{32}-\mathrm{i}\xi\varepsilon_1)}\{-k_nA_{2n}+\mathrm{i}\xi a_{33}A_{8n}+\mathrm{i}aa_{34}A_{6n}\},$$

$$A_{18n}=\frac{1}{(a_{32}-\mathrm{i}\xi\varepsilon_1)}\{\mathrm{i}aA_{2n}-\mathrm{i}\xi a_{33}A_{7n}-k_na_{34}A_{6n}\}.$$

The magnetic and the electric field intensities in a free space which can represented as

$$\frac{\partial h_0}{\partial y}=\varepsilon_1\frac{\partial E_{x0}}{\partial t},\qquad \frac{\partial h_0}{\partial x}=-\varepsilon_1\frac{\partial E_{y0}}{\partial t},\qquad \frac{\partial h_0}{\partial t}=\frac{\partial E_{x0}}{\partial y}-\frac{\partial E_{y0}}{\partial x}.\tag{52}$$

These variables can be assumed as the harmonic wave solution

$$[h_0,E_{x0},E_{x0}](x,y,t)=[h_0^*,E_{y0}^*,E_{y0}^*](y)e^{\mathrm{i}(ax-\xi t)},\tag{53}$$

then we obtain

$$[\mathrm{D}^2\text{-}a^2+\xi^2\varepsilon_1]\,h_0^*=0.\tag{54}$$

The solutions of the quantities h_0, E_{x0}, and E_{y0} are

$$h_0(x,y,t)=\alpha^*\exp\{(-k_8y)+\mathrm{i}(ax-\xi t)\},\tag{55}$$

$$E_{x0}(x,y,t)=\frac{k_8\alpha^*}{\mathrm{i}\xi\varepsilon_1}\exp\{(-k_8y)+\mathrm{i}(ax-\xi t)\},\tag{56}$$

$$E_{y0}(x,y,t)=\frac{a\alpha^*}{\xi\varepsilon_1}\exp\{(-k_ny)+\mathrm{i}(ax-\xi t)\},\tag{57}$$

where α^* is a constant and $k_8=\sqrt{a^2-\xi^2\varepsilon_1}$.

Take the dimensionless boundary conditions to get the constants $H_n\,(\mathrm{n}=1,2,\ldots,7)$ and ignore the positive exponentials at $y=0$,

$$\sigma_{yy}(x,0,t)=-p_1\exp(\mathrm{i}(ax-\xi t))-p. \qquad \sigma_{xy}(x,0,t)=0, \quad \text{and} \quad m_{yz}(x,0,t)=0. \tag{58}$$

$$T(x,0,t)=p_2\exp(\mathrm{i}(ax-\xi t)). \tag{59}$$

$$E_y(x,0,t)=E_{y0}(x,0,t), \text{ and } \quad h(0,y,t)=h_0(0,y,t). \tag{60}$$

$$\frac{\partial C(x,0,t)}{\partial y}=\frac{\partial \psi(x,0,t)}{\partial y}=0. \tag{61}$$

Substituting the expressions of the physical functions in equations (58)-(61) to get the values of the constants $H_n\,(\mathrm{n}=1,2,\ldots,7)$ and α^*.

3. NUMERICAL RESULTS AND DISCUSSION

A magnesium crystal-like material was chosen. The constants were taken as [54, 55]

$\lambda=9.4\times10^{10}\,N.m^{-2}$, $\mu=4\times10^{10}\,N.m^{-2}$, $k=1.7\times10^{2}\,N\,s^{-1}\cdot K^{-1}$, $\rho=1.74\times10^{3}\,kg.m^{-3}$, $\bar{k}=10^{10}\,N.m^{-2}$, $\alpha_t=7.4033\times10^{-7}\,K^{-1}$, $\alpha_c=2.65\times10^{-4}\,kg^{-1}m^{-3}$, $b=1.13849\times10^{10}\,N.m^{-2}$, $r=2\times10^{6}\,N.m^{-2}\cdot K^{-1}$, $\omega_0=0.0787\times10^{-3}\,N.m^{-2}\,s$, $a=0.03\,m$, $\varepsilon_0=10^{-9}/(36\pi)\,F.m^{-1}$, $\mu_0=4\pi\times10^{-7}\,H.m^{-1}$, $H_0=10^{8}\,A.m^{-1}$, $g=9.8\,m.s^{-2}$, $\sigma_0=9.36\times10^{5}\,Col.A/m$, $p_1=0.007\,K$, $t=8\times10^{-6}\,s$, $a_1=0.001\,rad/s$, $a_2=1.5\,rad/s$, $d=85\times10^{-10}\,kg.m^{-3}$, $a^*=2.9\times10^{4}\,m^2s^{-2}k^{-1}$, $b_1=32\times10^{5}\,m^5kg^{-1}s^{-2}$, $p^*=0.05\,N$, $C_E=1.04\times10^{3}\,J.kg^{-1}\cdot K^{-1}$, $\gamma=7.779\times10^{-8}\,N$, $J_0=2\times10^{-20}\,m^2$, $\chi=1.753\times10^{-15}\,m^2$, $\xi_1=1.475\times10^{10}\,N.m^{-2}$.

The adjective discussion clarifies the effect of the phase-lag parameters; [the heat flux τ_q and the gradient temperature τ_θ] in the case of two values for them. $\tau_q=0.006, 0.009\,s$ and $\tau_\theta=0.002, 0.003\,s$ in the behavior of the solution of the presented functions in 2-D figures (1-5).

First, we look at the behavior of the solution for two distinct values for both τ_q and τ_θ. The figures were obtained at a fixed all influential physical parameters in the problem. It can be seen that both of the phase-lag parameters τ_q and τ_θ have an observable and effectual role in the descriptive functions variation as they constrained to them by the increase or the decrease with the increase of τ_q and τ_θ.

Figure 2 clarifies that with the increase of the values of phase lag parameters τ_q and τ_θ the variation of the displacement components u and v are decreasing and increasing respectively. Figures 3 and 4 show the variation of σ_{xy} and m_{xy} are increasing while σ_{yy} and T are decreasing, figures 5 and 6 state that the variation of ψ and P are decreasing while the variation of ϕ_z and C are increasing with the increase of the values of phase lag parameters. It can observe that all functions studied are continuous and converge to its initial equilibrium states. The obtained results in the present study are in agreement with the results in previous published papers as in [5] for the gravity, as the similar results with [30] for the modified Ohm's law. Totally results of the present study were close to reference [41] for micropolar thermoelastic with vids due to dual-phase-lag.

Figures (7-10) are the 3-D representation of the damped propagation of the obtained physical functions of the studied medium. From all figures represented we can observe that all functions are propagated as the wave properties propagation.

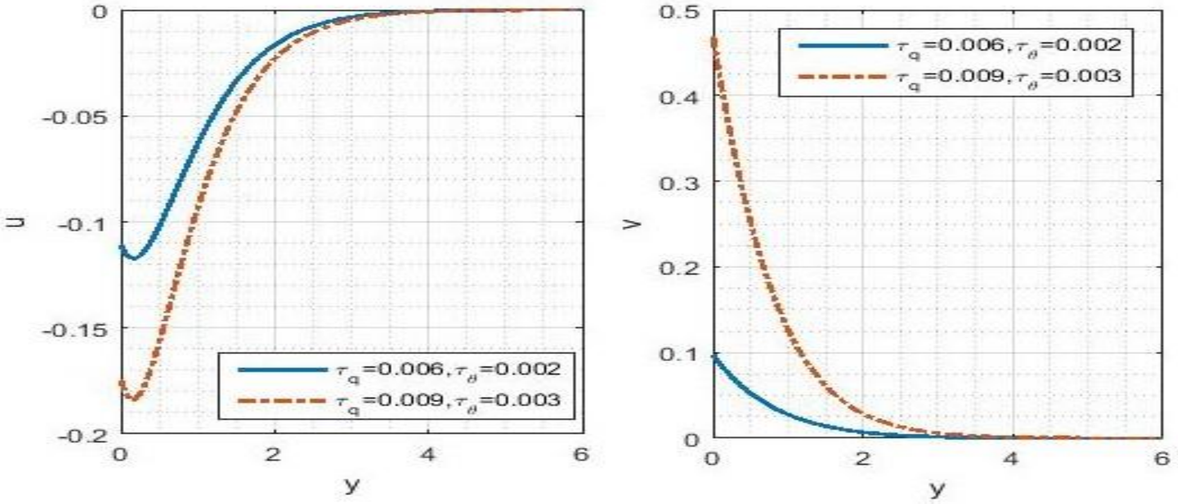

Fig. 2: Variation of τ_q and τ_θ in the distribution both of u and v.

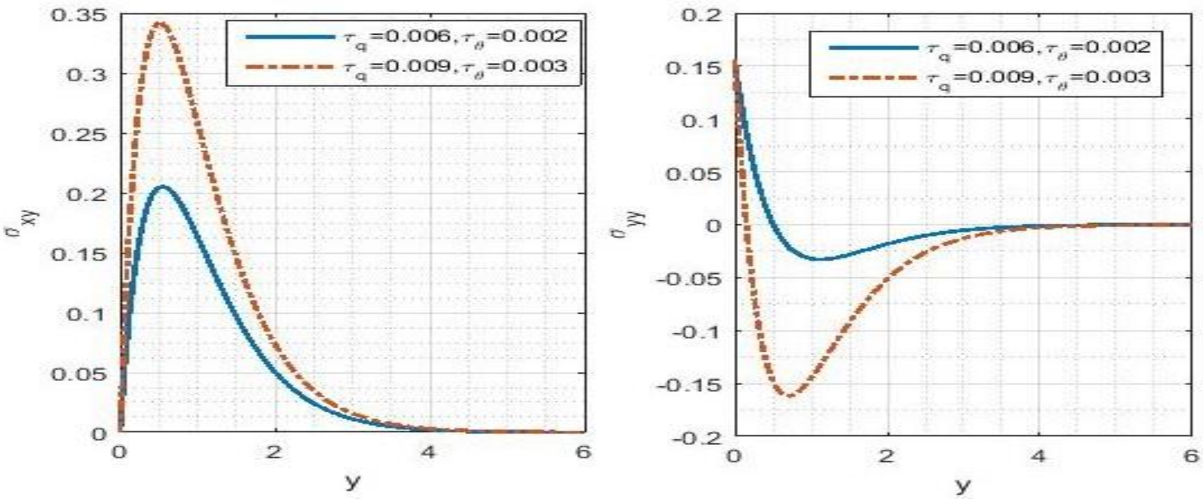

Fig. 3: Variation of τ_q and τ_θ in the distribution both of σ_{xy} and σ_{yy}.

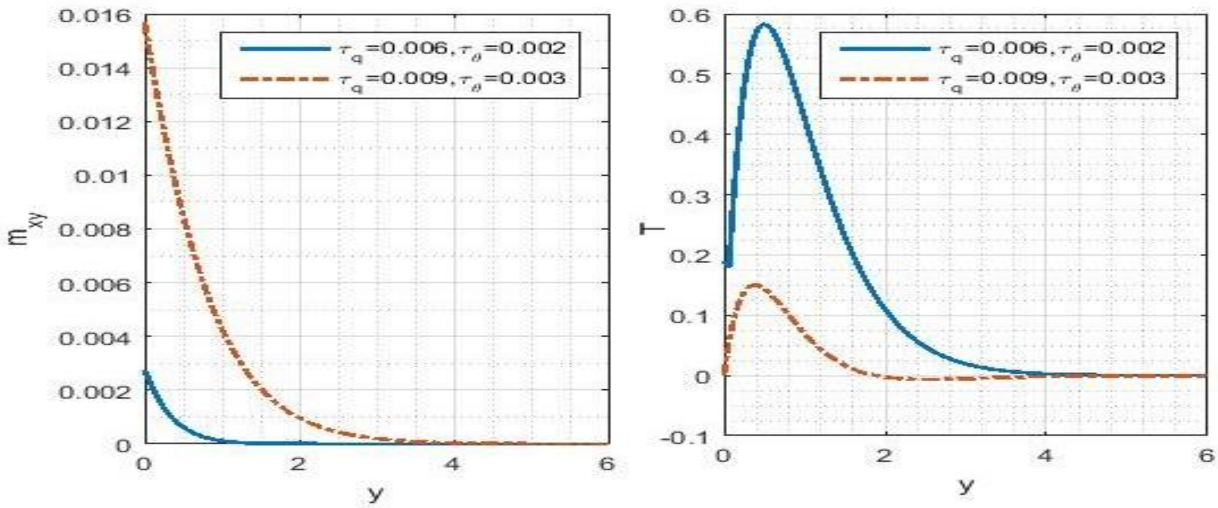

Fig. 4: Variation of τ_q and τ_θ in the distribution both of m_{xy} and T.

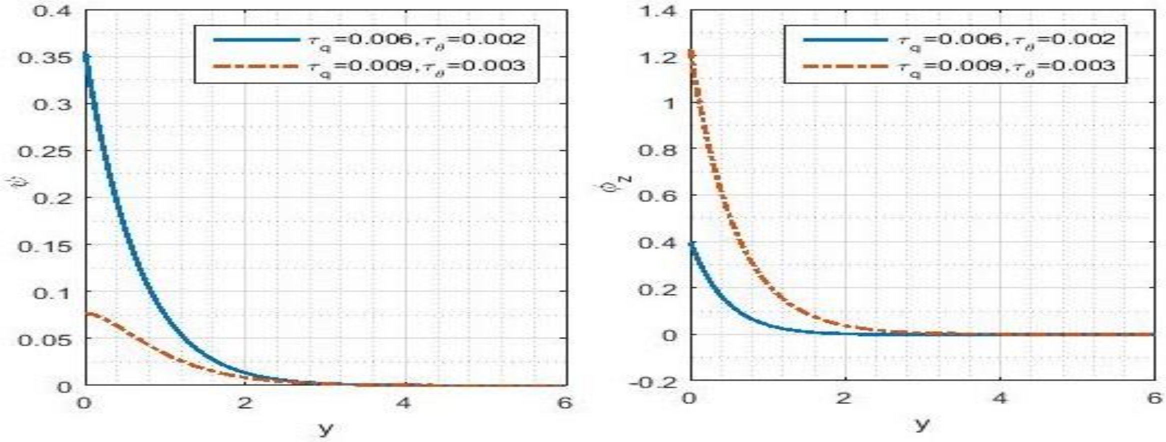

Fig. 5: Variation of τ_q and τ_θ in the distribution both of ψ and ϕ_z.

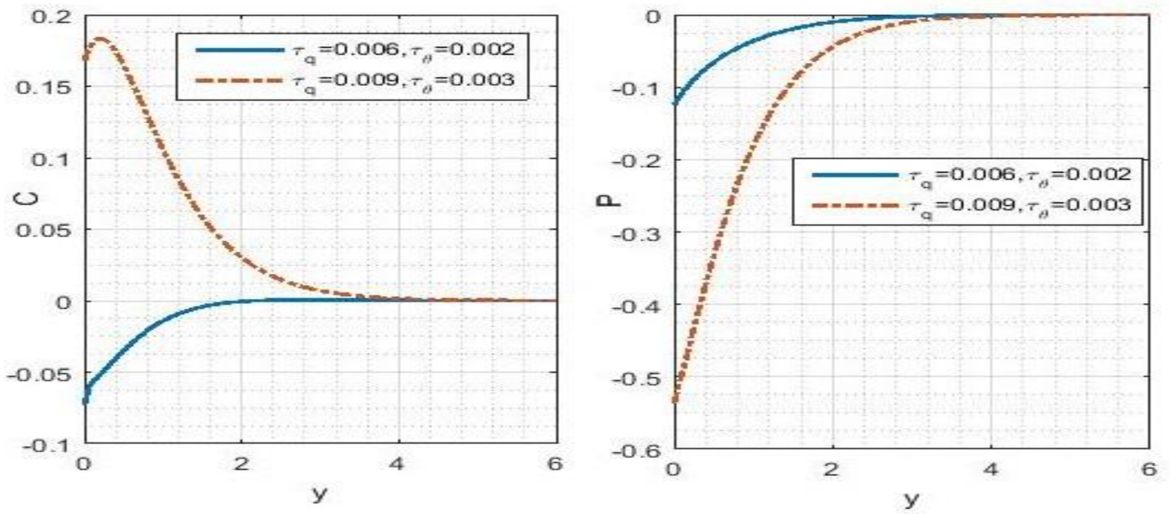

Fig. 6: Variation of τ_q and τ_θ in the distribution both of C and P.

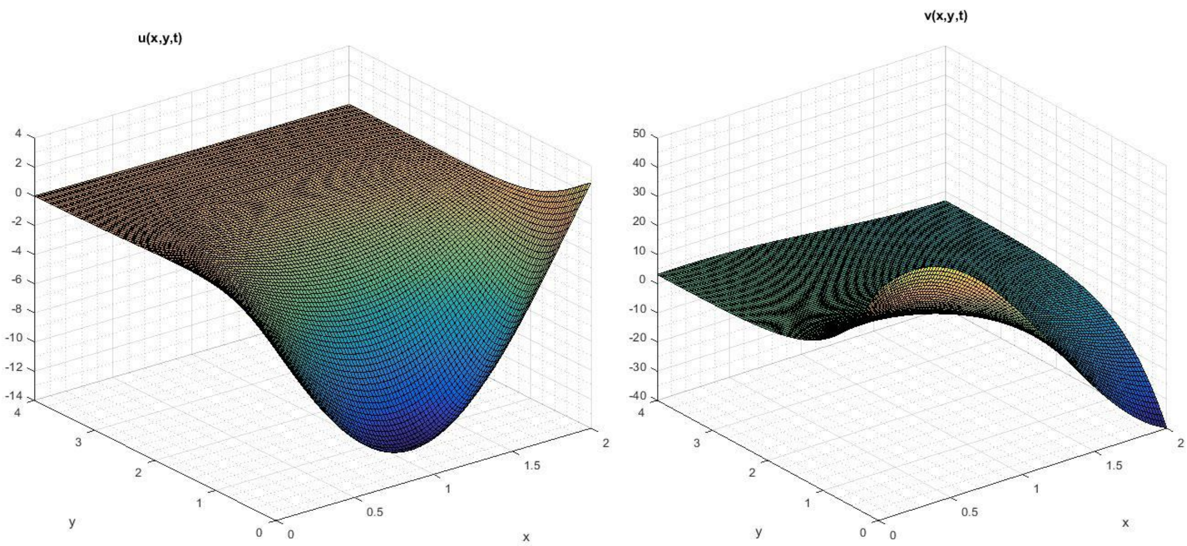

Fig. 7: Propagation of u and v wave in 3D.

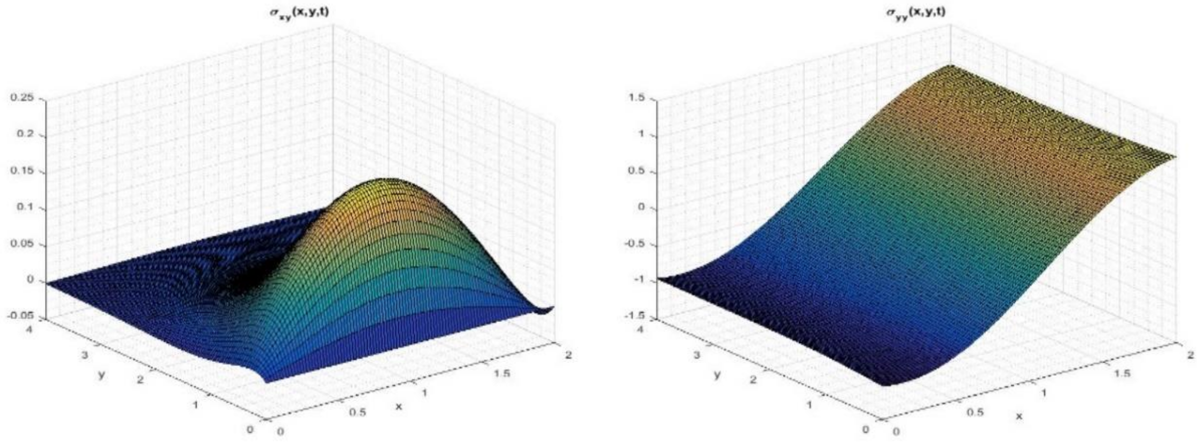

Fig. 8: Propagation of σ_{xy} and σ_{yy} in 3D.

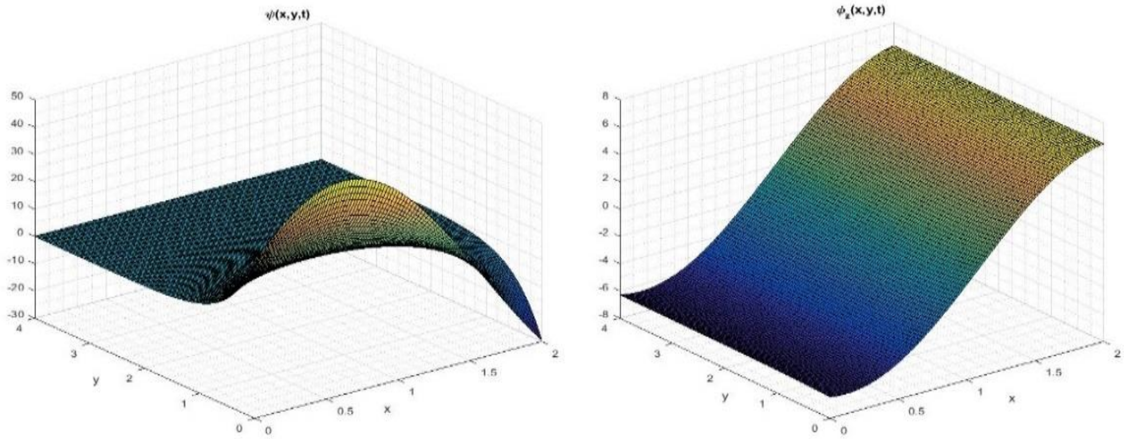

Fig. 9: Propagation of ψ and ϕ_z wave in 3D.

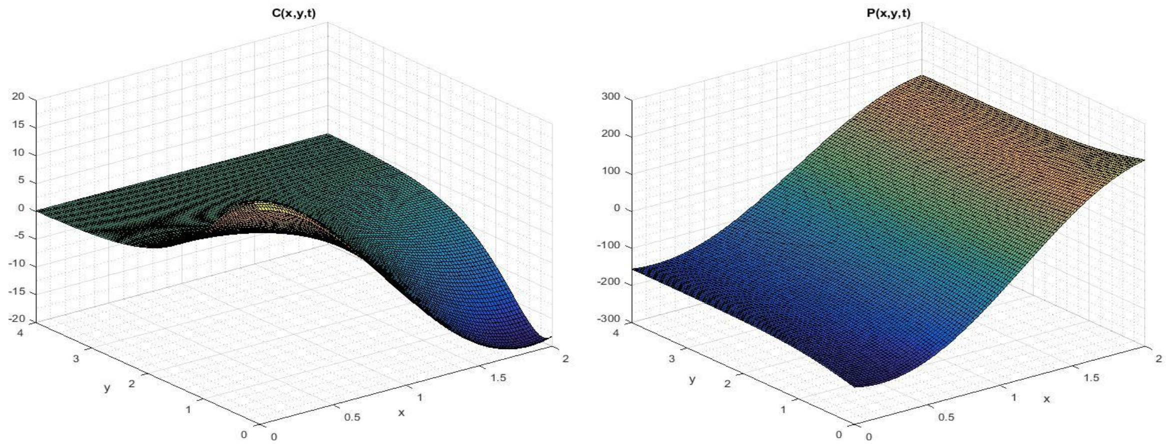

Fig. 10: Propagation of C and P wave in 3D.

4. CONCLUSIONS

The main conclusion of the presented problem is that the dual-phase-lag model is more realistic and effective in thermoelastic theory in its outcomes. All the used physical operators are more operative in the variation of the descriptive functions of the medium. The functions with their solutions are continuous and propagate as the wave solutions. The descriptive functions converge to their initial states. The current study can be applied in steel and geomechanics, as well as in earthquake engineering. The type of thermoelastic material utilized in this model is commonly applied in drilling operations and extensively in nuclear physics. The outcomes of all influences are substantial in the research.

CONFLICT OF INTEREST

No conflict of interest

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